(16) QN. → Discuss the convergence of the series

$$\frac{1}{1^p} + \frac{1}{3^p} + \frac{1}{5^p} + \frac{1}{7^p} + \dots \text{ to } \infty$$

for $p \in R$, the set of real numbers.

Ans. → The nth term of 1, 3, 5, 7 ....

$\therefore a = 1$

$d = 2$

$a + (n-1)d$

$1 + (n-1)2$

$1 + 2n - 2$

$2n - 1$

Let the nth term of the given series be denoted by u_n,

$$u_n = \frac{1}{(2n-1)^p} = \frac{1}{n^p\left(2-\frac{1}{n}\right)^p}$$

Let us consider an Auxiliary series whose nth term

$$v_n = \frac{1}{n^p}$$

$$\frac{u_n}{v_n} = \frac{\frac{1}{n^p\left(2-\frac{1}{n}\right)^p}}{\frac{1}{n^p}} = \frac{1}{\cancel{n^p}\left(2-\frac{1}{n}\right)^p} \times \frac{\cancel{n^p}}{1}$$

$$= \frac{1}{\left(2-\frac{1}{n}\right)^p}$$

$$\lim_{n\to\infty} \frac{u_n}{v_n} = \lim_{n\to\infty} \frac{1}{\left(2-\frac{1}{n}\right)^p} = \frac{1}{\left(2-\frac{1}{\infty}\right)^p} = \frac{1}{(2-0)^p} = \frac{1}{2^p}$$

$= \frac{1}{2^p}$ which is finite and non zero

$\therefore$ from comparison test $\sum u_n$ and $\sum v_n$ will convergent and divergent simultaneously.

But, $v_n = \frac{1}{n^p}$

$\therefore v_n$ is convergent $p > 1$

and v_n is divergent when, $p \leq 1$

Hence, the given series is convergent when, $p > 1$ and divergent $p \leq 1$.

(2) Q.No → Test the convergence of the series whose general term is

$$\frac{\sqrt{n+1}-\sqrt{n}}{n}$$

Ans. → Let the given nth term be denoted by u_n

$$u_n = \frac{\sqrt{n+1}-\sqrt{n}}{n} = \frac{\sqrt{n+1}-\sqrt{n}}{n} \cdot \frac{\sqrt{n+1}+\sqrt{n}}{\sqrt{n+1}+\sqrt{n}}$$

$$= \frac{\cancel{n}+1-\cancel{n}}{n(\sqrt{n+1}+\sqrt{n})} = \frac{1}{n \cdot \sqrt{n}\left(\sqrt{1+\frac{1}{n}}+1\right)}$$

$$= \frac{1}{n^{3/2}\left(\sqrt{1+\frac{1}{n}}+1\right)}$$

Let us consider an Auxiliary series Σv_n whose nth term $v_n = \frac{1}{n^{3/2}}$

$$\frac{u_n}{v_n} = \frac{\frac{1}{n^{3/2}\left(\sqrt{1+\frac{1}{n}}+1\right)}}{\frac{1}{n^{3/2}}} = \frac{1}{\cancel{n^{3/2}}\left(\sqrt{1+\frac{1}{n}}+1\right)} \times \frac{\cancel{n^{3/2}}}{1}$$

$$\lim_{n\to\infty} \frac{u_n}{v_n} = \lim_{n\to\infty} \frac{1}{\sqrt{1+\frac{1}{n}}+1} = \frac{1}{\sqrt{1+\frac{1}{\infty}}+1} = \frac{1}{\sqrt{1+0+1}}$$

$$= \frac{1}{\sqrt{2}} = \frac{1}{2}$$

$= \frac{1}{2}$ which is finite and non zero

∴ From Comparision test Σu_n and Σv_n will be convergent and divergent simultaneously

But, $v_n = \frac{1}{n^{3/2}}$ which is convergent

Hence, by comparison test,

Σu_n is also convergent.

(22) Q.No → Test the convergence of the series whose general term is

$$\frac{1}{\sqrt{1+n^2}+n}$$

Ans → Let the nth term be denoted by u_n

$$u_n = \frac{1}{\sqrt{1+n^2}+n} = \frac{1}{n\left(\sqrt{\frac{1}{n^2}+1}+1\right)}$$

Let us consider an Auxiliary series, whose nth term $v_n = \frac{1}{n}$

$$\frac{u_n}{v_n} = \frac{\frac{1}{n\left(\sqrt{\frac{1}{n^2}+1}+1\right)}}{\frac{1}{n}} = \frac{1}{\cancel{n}\left(\sqrt{\frac{1}{n^2}+1}+1\right)} \times \frac{\cancel{n}}{1}$$

$$\lim_{n\to\infty} \frac{u_n}{v_n} = \lim_{n\to\infty} \frac{1}{\left(\sqrt{\frac{1}{n^2}+1}+1\right)} = \frac{1}{\left(\sqrt{\frac{1}{\infty^2}+1}+1\right)} = \frac{1}{\left(\sqrt{0+1}+1\right)}$$

$= \frac{1}{\sqrt{2}} = \frac{1}{2}$ which is finite and non zero

∴ from comparision test Σu_n and Σv_n will be convergent and divergent simultaneously

~~But, $v_n = \frac{1}{n}$ which is finite and non zero~~

~~∴ from comparision test, Σu_n and Σv_n will be convergent and divergent~~

But, $v_n = \frac{1}{n}$ which is ~~convergent~~ divergent

Hence, by comparison test

Σu_n is also divergent.

(23) Q No → Test for Convergence the series whose general term is

$$\sqrt{n^2+1} - \sqrt{n^2-1}.$$

Ans. → Here, $u_n = \sqrt{n^2+1} - \sqrt{n^2-1}$

$$= \frac{(\sqrt{n^2+1} - \sqrt{n^2-1})(\sqrt{n^2+1} + \sqrt{n^2-1})}{\sqrt{n^2+1} + \sqrt{n^2-1}}$$

$$= \frac{\cancel{n^2}+1-\cancel{n^2}+1}{\sqrt{n^2+1} + \sqrt{n^2-1}} = \frac{2}{\sqrt{n^2+1} + \sqrt{n^2-1}}$$

$$= \frac{2}{n\left(\sqrt{1+\frac{1}{n^2}} + \sqrt{1-\frac{1}{n^2}}\right)}$$

Let us consider an Auxiliary series whose nth term $v_n = \frac{1}{n}$

$$\frac{u_n}{v_n} = \frac{\dfrac{2}{n\left(\sqrt{1+\frac{1}{n^2}} + \sqrt{1-\frac{1}{n^2}}\right)}}{\dfrac{1}{n}} = \frac{2}{\cancel{n}\left(\sqrt{1+\frac{1}{n^2}} + \sqrt{1-\frac{1}{n^2}}\right)} \times \frac{\cancel{n}}{1}$$

$$\underset{n\to\infty}{\text{Lt}} \frac{u_n}{v_n} = \underset{n\to\infty}{\text{Lt}} \frac{2}{\sqrt{1+\frac{1}{n^2}} + \sqrt{1-\frac{1}{n^2}}} = \frac{2}{\sqrt{1+\frac{1}{\infty^2}} + \sqrt{1-\frac{1}{\infty^2}}}$$

$$= \frac{2}{\sqrt{1+0} + \sqrt{1-0}} = \frac{\cancel{2}}{\cancel{2}} = 1$$ which is finite

∴ from Comparison test $\sum u_n$ and $\sum v_n$ will be Convergent and divergent simultaneously

But, $v_n = \frac{1}{n}$ which is divergent

Hence, by Comparison test

$\sum u_n$ is also divergent

24 Q No → Test for convergence the series whose nth term is

P.U. 03, P.U. 04, P.U. 02

$$\sqrt{n^3+1} - \sqrt{n^3-1}$$

Ans → Let the given nth term be denoted by

$$u_n = \sqrt{n^3+1} - \sqrt{n^3-1}$$

$$= \frac{(\sqrt{n^3+1} - \sqrt{n^3-1})(\sqrt{n^3+1} + \sqrt{n^3+1})}{\sqrt{n^3+1} + \sqrt{n^3-1}}$$

$$= \frac{\cancel{n^3} + 1 - \cancel{n^3} + 1}{\sqrt{n^3+1} + \sqrt{n^3-1}} = \frac{2}{n^{3/2}\left[\sqrt{1+\frac{1}{n^3}} + \sqrt{1-\frac{1}{n^3}}\right]}$$

Let us consider an Auxiliary series whose nth term $v_n = \frac{1}{n^{3/2}}$

$$\frac{u_n}{v_n} = \frac{\dfrac{2}{n^{3/2}\left[\sqrt{1+\frac{1}{n^3}} + \sqrt{1-\frac{1}{n^3}}\right]}}{\dfrac{1}{n^{3/2}}} = \frac{2}{\cancel{n^{3/2}}\left[\sqrt{1+\frac{1}{n^3}} + \sqrt{1-\frac{1}{n^3}}\right]} \times \frac{\cancel{n^{3/2}}}{1}$$

$$\lim_{n\to\infty} \frac{u_n}{v_n} = \lim_{n\to\infty} \frac{2}{\sqrt{1+\frac{1}{n^3}} + \sqrt{1-\frac{1}{n^3}}} = \frac{2}{\sqrt{1+\frac{1}{\infty^3}} + \sqrt{1-\frac{1}{\infty^3}}}$$

$$= \frac{2}{\sqrt{1+0} + \sqrt{1-0}} = \frac{\cancel{2}}{\cancel{2}} = 1$$

∴ from comparision test Σu_n and Σv_n will be convergent and divergent simultaneously

But $v_n = \frac{1}{n^{3/2}}$ which is convergent

Σu_n is also convergent